GRADE 10 MATHEMATICS NOTES

COMPLETE FOR TERM 1, 2, AND 3

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STRAND 1.0: NUMBERS AND ALGEBRA

Introduction to the Strand

This strand lays the foundation for advanced mathematical thinking in Senior School. It builds upon the concepts learned in Junior School, deepening the understanding of the number system, the power of indices, the utility of logarithms in calculation, and the algebraic manipulation of quadratic expressions. These notes are designed to be a comprehensive guide for learners and teachers, offering detailed explanations, examples, and practical tips.

SUB STRAND 1.1: REAL NUMBERS

Overview In this sub-strand, we move beyond simple counting. We explore the nature of numbers, how they are categorized, and how to manipulate them using the concept of reciprocals. Understanding the "personality" of numbers is crucial for algebra and higher-level mathematics.

Classification of Whole Numbers

Whole numbers are the set of numbers {0, 1, 2, 3, ...}. However, within this set, numbers have different properties. We classify them into four distinct categories based on divisibility and factors.

- Odd Numbers
  - Definition: An odd number is any integer that cannot be divided exactly by 2. When divided by 2, it leaves a remainder of 1.

  - General Form: 2n + 1 (where n is an integer).
  - Examples: 1, 3, 5, 7, 9, 11, 15, 101.
  - Tip: To check if a massive number is odd, just look at the last digit. If it ends in 1, 3, 5, 7, or 9, the whole number is odd.
- Even Numbers
  - Definition: An even number is an integer that is exactly divisible by 2 with no remainder.
  - General Form: 2n (where n is an integer).
  - Examples: 0, 2, 4, 6, 8, 10, 24, 100.
  - Tip: Any number ending in 0, 2, 4, 6, or 8 is even. Zero is considered an even number because it sits between -1 (odd) and 1 (odd) and is divisible by 2 ($0 \div 2 = 0$).
- Prime Numbers
  - Definition: A prime number is a whole number greater than 1 that has exactly two factors: 1 and itself.
  - Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
  - Important Note: The number 2 is the only even prime number. All other prime numbers are odd (because any other even number is divisible by 2).
  - The number 1 is NOT a prime number. This is a common mistake. It is not prime because it has only one factor (itself), not two.
- Composite Numbers

- Definition: A composite number is a positive integer that has at least one factor other than 1 and itself. Essentially, if a number (greater than 1) is not prime, it is composite.
- Examples: 4 (factors: 1, 2, 4), 6 (factors: 1, 2, 3, 6), 9 (factors: 1, 3, 9).
- Tip: You can think of composite numbers as "rectangles" because they can be arranged into rectangular arrays (e.g., 6 dots can be 2 rows of 3), whereas prime numbers are just "lines."

Classification of Real Numbers

The Real Number system includes every number you can find on a continuous number line. We split these into Rational and Irrational numbers.

- Rational Numbers (Q)
  - Definition: A rational number is any number that can be expressed as a fraction p/q, where p and q are integers and q is not equal to zero.
  - Includes:
    - Integers: 5 (can be written as 5/1), -3 (-3/1).
    - Terminating Decimals: 0.5 (1/2), 0.125 (1/8).
    - Recurring (Repeating) Decimals: 0.333... (1/3), 0.142857... (1/7).
  - Tip: If you can write it as a simple fraction, it is rational.
- Irrational Numbers (I)
  - Definition: An irrational number cannot be written as a simple fraction. Its decimal expansion goes on forever without terminating and without repeating a pattern.

- Examples:
  - √2 (approx 1.4142...): The square root of any non-perfect square is irrational (called a surd).
  - √3, √5, √7.
  - π (Pi): The ratio of a circle's circumference to its diameter (approx 3.14159...).
  - e (Euler's number): Approx 2.718...
- Note: While 22/7 is often used as an approximation for Pi, 22/7 is Rational, while Pi itself is Irrational.

Reciprocals of Real Numbers

The reciprocal of a number is what you multiply the number by to get 1. It is also known as the "multiplicative inverse."

- Definition: The reciprocal of a number 'x' is 1/x.
- Example: The reciprocal of 5 is 1/5 (or 0.2). The reciprocal of 2/3 is 3/2.

Method A: Determining Reciprocals by Division

- This is the fundamental method. You simply divide 1 by the number.
- Example: Find the reciprocal of 8.
  - Calculation: 1 ÷ 8 = 0.125.
- Example: Find the reciprocal of 0.25.
  - Calculation: 1 ÷ 0.25 = 1 ÷ (1/4) = 4.

Method B: Using Mathematical Tables In examinations and classroom settings without calculators, four-figure mathematical tables are used. This requires specific skills in reading the table.

- Standard Form Requirement: To use reciprocal tables, it is often easier if the number is first converted to standard form (scientific notation), though most tables list numbers from 1.0 to 9.9.
- How to read the table for a number like 3.456:
    1. Locate "3.4" in the left-hand column (x).
    2. Move across to the main column headed "5". Write down this value (e.g., 0.2899).
    3. Continue across to the "Mean Differences" (Add) columns and look under "6". Note this value (e.g., 5).
    4. Subtract the mean difference from the last digits of the main value. Note: Unlike logarithms where we add, for reciprocals, we usually subtract the difference because as a number gets bigger, its reciprocal gets smaller. *Check your specific table's instructions; most reciprocal tables require subtraction of the mean difference.*
- Handling Decimals:
    - To find the reciprocal of 34.56:
    - Write 34.56 as 3.456×10^1.
    - Reciprocal = $1 / (3.456 \times 10^1) = (1/3.456) \times 10^{-1}$.
    - Find reciprocal of 3.456 from tables (0.2894).
    - Result: $0.2894 \times 10^{-1} = 0.02894$.

Method C: Using Calculators

- This is the most direct method.
- Steps:
    1. Type the number.
    2. Press the button labeled [x^{-1}] or [1/x].
    3. Press [=].
- Tip: Always check if the answer makes sense. The reciprocal of a large number should be very small, and the reciprocal of a decimal less than 1 should be greater than 1.

4. Applications of Reciprocals in Computations Reciprocals are primarily used to simplify division. Dividing by a number is the same as multiplying by its reciprocal.

- Example: Calculate 25 ÷ 0.125
- Instead of long division, recognize that 0.125 = 1/8.
- The reciprocal of 1/8 is 8.
- Calculation becomes: 25 × 8 = 200.
- This is very useful in physics formulas (like parallel resistors: 1/R = 1/R1 + 1/R2) and lens equations.

Suggested Learning Activities for Sub Strand 1.1

Activity 1: The Number Sort

- Prepare cards with various numbers: 0, 1, 2, 9, 11, 15, 0.333..., √2, π, 22/7, -5.

- Draw two large circles on the floor or whiteboard: "Rational" and "Irrational." Inside Rational, place "Integers," and inside Integers, place "Whole Numbers."
- Have students pick a card and stand in the correct zone. Ask them to justify why they are standing there.

Activity 2: Reciprocal Relay

- Divide the class into teams.
- Write a list of numbers on the board (e.g., 2, 4, 10, 0.5, 0.1).
- One student from each team runs to the board, writes the reciprocal in decimal form next to the number, and runs back to tag the next student.
- Variation: Use mathematical tables vs. Calculators to see which is faster for specific numbers.

SUB STRAND 1.2: INDICES AND LOGARITHMS

Overview Indices (exponents) allow us to write very large or very small numbers efficiently. Logarithms are the "inverse" of indices. Before calculators were powerful, logarithms were the only way to perform complex multiplications and root calculations. Today, they remain essential in calculus, science (pH scale, Richter scale), and computer science.

Indices (Exponents)

An index (plural: indices) tells you how many times to multiply a number (the base) by itself.

- Format: a^n
  - 'a' is the Base.
  - 'n' is the Index (or exponent/power).
- Example: $5^3 = 5 \times 5 \times 5 = 125$.

2. The Laws of Indices To handle expressions involving indices, we rely on standard laws. These apply when the bases are the same.

Law 1: Multiplication Law

- Rule: $a^m \times a^n = a^{m+n}$
- Explanation: When multiplying terms with the same base, add the powers.
- Example: $2^3 \times 2^4 = 2^{3+4} = 2^7$.

Law 2: Division Law

- Rule: $a^m \div a^n = a^{m-n}$

- Explanation: When dividing terms with the same base, subtract the power of the denominator from the power of the numerator.
- Example: $y^8 \div y^2 = y^{8-2} = y^6$.

Law 3: Power of a Power Law

- Rule: $(a^m)^n = a^{mxn}$
- Explanation: When raising a power to another power, multiply the indices.
- Example: $(3^2)^3 = 3^{2x3} = 3^6$.

Law 4: The Zero Index

- Rule: $a^0 = 1$ (provided $a \neq 0$)
- Explanation: Any non-zero number raised to the power of zero is 1.
- Why? Consider $5^2 \div 5^2$. By division law, it is $5^{2-2} = 5^0$. By arithmetic, $25 \div 25 = 1$. Therefore, $5^0 = 1$.

Law 5: Negative Indices

- Rule: $a^{-n} = 1 / a^n$
- Explanation: A negative index indicates a reciprocal.
- Example: $2^{-3} = 1 / 2^3 = 1 / 8$.
- Example: $(2/3)^{-2} = (3/2)^2 = 9/4$. (The fraction flips).

Law 6: Fractional Indices (Roots)

- Rule: $a^{1/n} = \sqrt[n]{a}$

- Rule: $a^{m/n} = \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$
- Explanation: The denominator of the fraction is the root; the numerator is the power.
- Example: $8^{1/3} = \sqrt[3]{8} = 2$.
- Example: $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$.

3. Logarithms to Base 10 Logarithms are simply indices written in a different format.

- Definition: If $a^n = x$, then $\log_a x = n$.
- "Log to the base 'a' of 'x' is equal to 'n'."
- This asks the question: "To what power must we raise base 'a' to get 'x'?"

Common Logarithms

- Logarithms with base 10 are called "common logarithms."
- If you see "log x" without a base written, it usually means base 10.
- Examples:
  - Since $10^2 = 100$, then $\log_{10} 100 = 2$.
  - Since $10^3 = 1000$, then $\log_{10} 1000 = 3$.
  - Since $10^{-1} = 0.1$, then $\log_{10} 0.1 = -1$.

4. Determining Common Logarithms For numbers that are not perfect powers of 10 (like 35 or 0.42), we use mathematical tables or calculators.

Using Mathematical Tables (Log Tables)

A log value has two parts:

1. The Characteristic: The integer part (determined by inspection).

2. The Mantissa: The decimal part (found in the tables).

- Step 1: Convert the number to Standard Form ($A \times 10^n$).
    - The power 'n' becomes the Characteristic.
- Step 2: Look up 'A' in the log tables to find the Mantissa.
    - The mantissa is always positive.

Example 1: Find log 452 using tables.

1. Standard Form: 4.52×10^2
2. Characteristic: 2
3. Mantissa: Look up 4.5 in the main column, move to column 2. Let's say the value is 0.6551.
4. Answer: 2.6551

Example 2: Find log 0.0342 using tables.

1. Standard Form: 3.42×10^{-2}
2. Characteristic: -2. In logs, we write this as "bar 2" (2 with a bar on top) to indicate only the 2 is negative, not the decimal part. Write: bar 2.
3. Mantissa: Look up 3.4 under column 2. Let's say it is 0.5340.
4. Answer: bar 2.5340. (This means -2 + 0.5340).
5. Laws of Logarithms Just like indices, logarithms have laws used for simplifying expressions.

- Law 1 (Product): log (A × B) = log A + log B

  - Multiplication inside the log becomes addition outside.
- Law 2 (Quotient): log (A ÷ B) = log A - log B
  - Division inside the log becomes subtraction outside.
- Law 3 (Power): log (A^n) = n log A
  - The power drops down to the front.

6. Application: Multiplication, Division, Powers, and Roots Before calculators, this was the primary use of logs. We can perform difficult math by converting numbers to logs, adding/subtracting them, and then converting back (Antilog).

Procedure for Calculation:

1. Create a table with columns: Number, Standard Form, Log.
2. Find the log of each number involved in the calculation.
3. Apply operations:
   - If multiplying numbers -> Add their logs.
   - If dividing numbers -> Subtract their logs.
   - If raising to a power -> Multiply the log by the power.
   - If taking a root -> Divide the log by the root index.
4. Find the Antilog of the final result to get the actual answer.

Example: Calculate √(35.2 × 4.1)

1. Find log 35.2 = 1.5465 (approx)
2. Find log 4.1 = 0.6128 (approx)
3. Add them (multiplication rule): 1.5465 + 0.6128 = 2.1593

4. Divide by 2 (square root rule): 2.1593 ÷ 2 = 1.0796
5. Find Antilog of 1.0796.
    - Look up 0.0796 in Antilog tables.
    - Apply the characteristic (1 means $\times 10^1$).
    - Result approx 12.01.

Suggested Learning Activities for Sub Strand 1.2

Activity 1: Index Wars (Card Game)

- Create a deck of cards. Half have index expressions (e.g., $2^3 \times 2^2$) and the other half have the simplified forms (e.g., 2^5).
- Students play "Go Fish" or "Memory" to match the problem with the simplified answer.

Activity 2: The Logarithmic Ladder

- Draw a ladder on the board. At the bottom, put simple numbers (10, 100, 1000). At the top, put complex decimals.
- Students must "climb" the ladder by converting the numbers to Log form correctly. The first student to reach the top with correct characteristics and mantissas wins.

SUB STRAND 1.3: QUADRATIC EXPRESSIONS AND EQUATIONS 1

Overview

This sub-strand introduces non-linear algebra. While linear equations (like $y = mx + c$) produce straight lines, quadratic equations produce curves (parabolas). This mathematics is essential for describing gravity, area, profit maximization, and projectile motion.

Quadratic Expressions

A quadratic expression is an algebraic expression where the highest power of the variable is 2.

- General Form: $ax^2 + bx + c$
  - 'a' is the coefficient of x^2 (a cannot be 0).
  - 'b' is the coefficient of x.
  - 'c' is the constant term.
- Examples:
  - $x^2 + 5x + 6$
  - $3y^2 - 9$
  - x^2 (here b=0, c=0)

Quadratic Identities

Identities are equations that are true for ALL values of the variables. There are three major quadratic identities you must memorize and understand.

Identity 1: The Square of a Sum

- $(a + b)^2 = a^2 + 2ab + b^2$
- Derivation from Area: Imagine a square with side length (a+b).
  - The area is (a+b)(a+b).
  - Inside, you can split it into a big square (a^2), a small square (b^2), and two rectangles (ab and ab).
  - Total Area = $a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

Identity 2: The Square of a Difference

- $(a - b)^2 = a^2 - 2ab + b^2$
- Tip: Watch the negative sign. It only affects the middle term.

Identity 3: The Difference of Two Squares (DOTS)

- $(a + b)(a - b) = a^2 - b^2$
- This is crucial for factorization. If you see a square minus another square, it always factors into (sum)(difference).
- Example: $x^2 - 9 = (x + 3)(x - 3)$.

3. Factorization of Quadratic Expressions Factorization is the reverse of expansion. We want to put the expression back into brackets.

Method A: Common Factors

- Look for a number or variable common to all terms.
- Example: $2x^2 + 4x = 2x(x + 2)$.

Method B: Factorization by Grouping (The AC Method) Used for expressions like $ax^2 + bx + c$.

1. Identify a, b, and c.
2. Find the Product (P) = a × c.
3. Find the Sum (S) = b.
4. Find two numbers (factors) that multiply to give P and add to give S.
5. Rewrite the middle term (bx) using these two numbers.
6. Factor by grouping.

- Example: Factorize $x^2 + 5x + 6$
  - a=1, b=5, c=6.
  - Product = 6. Sum = 5.
  - Factors of 6: 1&6, 2&3. Which add to 5? 2 and 3.
  - Rewrite: $x^2 + 2x + 3x + 6$
  - Group: $(x^2 + 2x) + (3x + 6)$
  - Factor groups: $x(x + 2) + 3(x + 2)$
  - Final Answer: $(x + 3)(x + 2)$.

Forming Quadratic Equations

We can form equations from word problems or geometric situations.

- Scenario: "The area of a rectangle is 24 cm^2. The length is 2 cm more than the width."
- Let width = x.
- Length = x + 2.
- Area = width × length = $x(x + 2)$.

- Equation: $x(x + 2) = 24 \rightarrow x^2 + 2x - 24 = 0$.

5. Solving Quadratic Equations by Factorization An expression becomes an equation when we set it equal to something (usually 0).

- Zero Product Property: If $A \times B = 0$, then either $A = 0$ or $B = 0$.
- This is why we factorize!

Steps to Solve:

1. Ensure the equation equals zero (e.g., $ax^2 + bx + c = 0$).
2. Factorize the quadratic expression.
3. Set each bracket equal to zero.
4. Solve for x.

Example: Solve $x^2 - 5x + 6 = 0$

1. Factorize: $(x - 2)(x - 3) = 0$.
2. Case 1: $x - 2 = 0 \rightarrow x = 2$.
3. Case 2: $x - 3 = 0 \rightarrow x = 3$.
4. Solutions: $x = 2$ or $x = 3$.
5. Application to Real Life Situations When solving word problems, you may get two answers (e.g., $x = 5$ and $x = -3$). You must interpret them.

- If 'x' represents a length or time, it cannot be negative. You would discard the negative value (-3) and state the answer is 5.
- Context is key in quadratic applications.

Suggested Learning Activities for Sub Strand 1.3

Activity 1: Algebra Tile Puzzles

- Provide students with physical or paper "Algebra Tiles" (Squares for x^2, rectangles for x, small squares for 1).
- Ask them to build rectangles representing expressions like $x^2 + 4x + 3$.
- The dimensions of the rectangle they build represent the factors (x+1) and (x+3).

Activity 2: The Projectile Plot

- Go outside. Have a student throw a ball in an arc.
- Explain that the path is a parabola (quadratic).
- Back in class, provide data points of height vs. time and ask students to determine when the ball hits the ground (solving for h=0).

Activity 3: Loop Card Game (Dominoes)

- Create cards where the right side has a quadratic equation ($x^2 - 9$) and the left side of the next card has the factors (x+3)(x-3).
- Students must link them all together in a continuous loop.

Summary of Key Formulas for Strand 1

- Reciprocal: $1/x$
- Indices: $a^m \times a^n = a^{m+n}$
- Indices: $a^{-n} = 1/a^n$
- Logarithms: $\log(ab) = \log a + \log b$
- Quadratic Identity: $(a+b)^2 = a^2 + 2ab + b^2$
- Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

Tips for Success in Strand 1

- Practice Mental Math: Try to identify prime numbers and simple squares (1-15) in your head.
- Be Organized: When using log tables, keep your columns straight. A messy table leads to calculation errors.
- Check Signs: In quadratics, the signs (+ or -) in the brackets determine everything. Always expand your answer back to check if you get the original expression.
- Show Working: In exams, marks are awarded for the method (M1) even if the final answer (A1) is wrong due to a small slip.

This concludes the detailed notes for Strand 1.0. These concepts are the building blocks for the Geometry and Statistics strands that follow.

STRAND 2.0: MEASUREMENTS AND GEOMETRY

Introduction to the Strand

This strand delves into the spatial aspects of mathematics. It moves from 2D shapes and their transformations to 3D solids and their properties, and finally into the study of motion and vectors. The goal is to develop spatial reasoning, understand the physical properties of objects (like area and volume), and quantify movement. These notes provide a comprehensive guide for learners, covering every sub-strand detailed in the curriculum design.

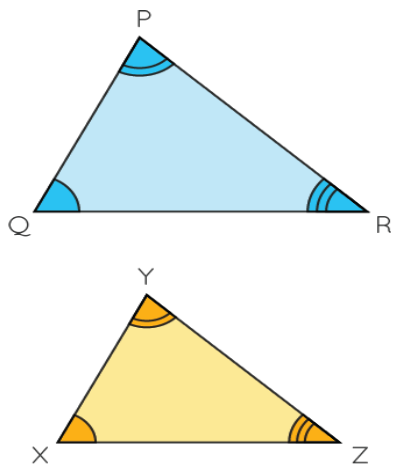
SUB STRAND 2.1: SIMILARITY AND ENLARGEMENT

Overview

This sub-strand explores how shapes can change size while maintaining their proportions. It connects geometry with algebra through the use of scale factors. Understanding similarity is crucial for architecture, engineering, and map reading.

Concept of Similarity

- **Definition**: Two figures are "similar" if they have the same shape but not necessarily the same size.



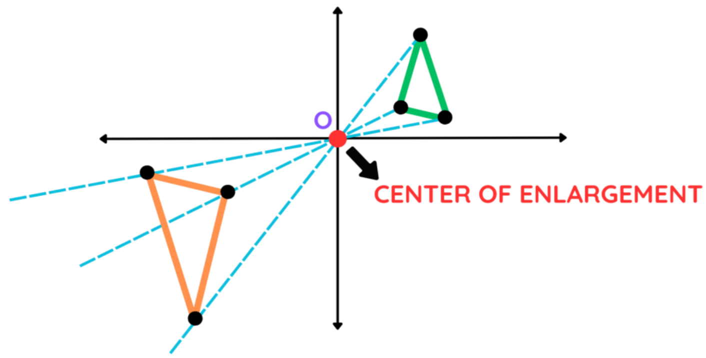
ΔQPR ~ ΔXYZ

- **Conditions for Similarity**:
  - **Corresponding Angles**: Must be equal. If you zoom in on a photo, the angles of the objects inside don't change.
  - **Corresponding Sides**: Must be in the same ratio. If one side doubles in length, all corresponding sides must double.
- **Visualizing Similarity**: Think of a model car and a real car. They look identical in shape, but one is a scaled-down version of the other.

Enlargement

Enlargement is a transformation that changes the size of an object. Despite the name, "enlargement" in mathematics can also mean making something smaller (reduction).

- **Key Elements of Enlargement**:
  - **Centre of Enlargement**: The fixed point from which the enlargement happens. Lines connecting corresponding points on the object and the image all meet at this center.



- **Linear Scale Factor (k)**: The ratio that determines how much the object grows or shrinks.

- **The Linear Scale Factor (k)**
  - **Formula**: k = Length of Image Side / Length of Corresponding Object Side.
  - **Interpreting k**:
    - If $k > 1$: The image is larger than the object (Magnification).
    - If $0 < k < 1$: The image is smaller than the object (Diminution or Reduction).
    - If $k = 1$: The image is the same size as the object (Congruent).
    - If k is negative: The image is on the opposite side of the center of enlargement and is inverted (flipped).

Constructing Enlargements

- **Given the Center and Scale Factor**:
  1. Draw lines (rays) from the Center of Enlargement through each vertex (corner) of the object.
  2. Measure the distance from the Center to a vertex.

3. Multiply this distance by the Scale Factor (k).
4. Measure the new distance along the ray to mark the new vertex.
5. Connect the new vertices to form the image.

- **Finding the Center of Enlargement**:

1. Draw a line connecting a point on the object to its corresponding point on the image.
2. Repeat this for another pair of points.
3. The point where these lines intersect is the Center of Enlargement.

Area and Volume Scale Factors

When you scale the lengths of an object, the area and volume change much faster.

- **Linear Scale Factor (L.S.F)**: The ratio of lengths = k.
- **Area Scale Factor (A.S.F)**: The ratio of areas.
  - **Relationship**: A.S.F = $(L.S.F)^2$
  - **Example**: If you double the sides of a square (L.S.F = 2), the area becomes 4 times larger ($2^2 = 4$).
- **Volume Scale Factor (V.S.F)**: The ratio of volumes.
  - **Relationship**: V.S.F = $(L.S.F)^3$
  - **Example**: If you double the radius of a sphere (L.S.F = 2), the volume becomes 8 times larger ($2^3 = 8$).
- **Connecting the Three**:
  - If you know the A.S.F, take the square root to find the L.S.F.
  - If you know the V.S.F, take the cube root to find the L.S.F.

- **Tip**: Always convert back to L.S.F before moving between Area and Volume. Do not try to go directly from Area to Volume without finding the Linear Scale Factor first.

Applications in Real Life

- **Maps and Models**: Calculating actual distances from maps or building physical models of buildings.
- **Photography**: Resizing images for print or digital screens.
- **Biology**: Comparing the surface area to volume ratios of cells (why cells are small).

Suggested Learning Activities for 2.1

- **The Shadow Activity**: Take a flashlight and an object (like a cutout shape). Project a shadow onto a wall. Measure the object and the shadow. Calculate the scale factor. Move the object closer to the light and observe how the scale factor changes.
- **Model Building**: Create a scale model of the classroom using a cardboard box. Students must measure the room and decide on a scale factor (e.g., 1:50) to make the furniture fit.

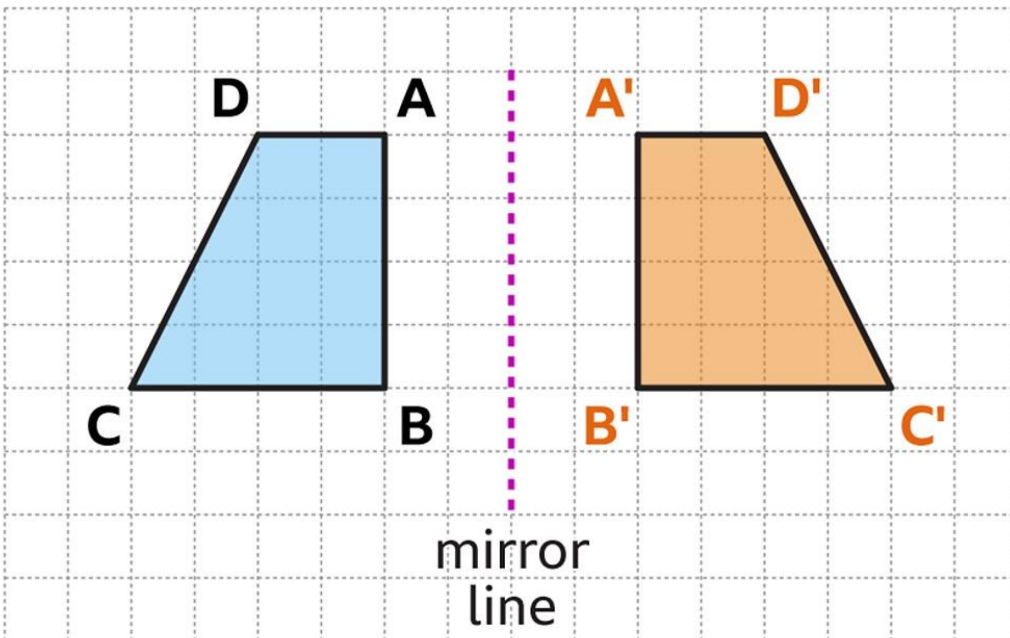
SUB STRAND 2.2: REFLECTION AND CONGRUENCE

Overview

This sub-strand focuses on symmetry and exactness. Reflection is a "flip" over a line, while congruence deals with shapes that are identical in size and shape.

Reflection

Reflection is a transformation where an object is flipped across a line, creating a mirror image.



- **Properties of Reflection**:
  - **Equidistant**: The object and the image are the same distance from the mirror line.
  - **Perpendicular**: The line connecting a point and its image is perpendicular (at 90 degrees) to the mirror line.
  - **Lateral Inversion**: The image is laterally inverted (left becomes right).

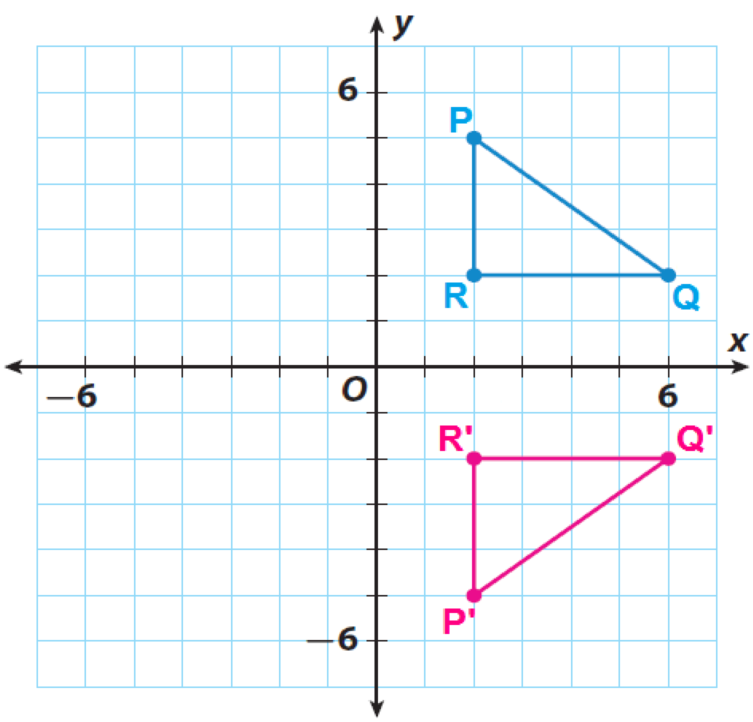
- **Size**: The image is the same size as the object (Isometric).

Lines of Symmetry

- **Plane Figures**: A line of symmetry divides a shape into two identical halves that are mirror images of each other.
  - Square: 4 lines of symmetry.
  - Rectangle: 2 lines of symmetry.
  - Equilateral Triangle: 3 lines of symmetry.
  - Circle: Infinite lines of symmetry.

Reflection on the Cartesian Plane

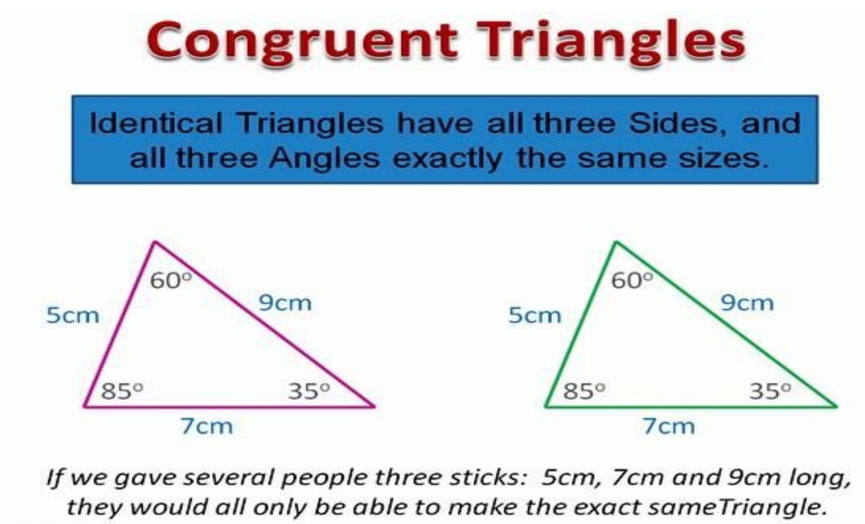
We often perform reflections using coordinate geometry.



- **Reflection in the x-axis (y = 0)**: The x-coordinate stays the same, the y-coordinate changes sign. Point (x, y) becomes (x, -y).
- **Reflection in the y-axis (x = 0)**: The y-coordinate stays the same, the x-coordinate changes sign. Point (x, y) becomes (-x, y).
- **Reflection in the line y = x**: The coordinates swap. Point (x, y) becomes (y, x).
- **Reflection in the line y = -x**: The coordinates swap and change signs. Point (x, y) becomes (-y, -x).
- **Finding the Mirror Line**:
  - If you have an object and its image, connect two corresponding points.
  - Find the midpoint of that connecting line.
  - The mirror line passes through that midpoint and is perpendicular to the connecting line.

Congruence

Two figures are congruent if they are identical in shape and size. If you cut one out, it would fit perfectly on top of the other.



- **Direct vs. Opposite Congruence**:
    - **Direct Congruence**: You can fit one shape onto the other just by rotating or moving it (Translation/Rotation).
    - **Opposite Congruence**: You must flip the shape to make it fit (Reflection).

Congruence Tests for Triangles

To prove two triangles are congruent, you don't need to measure everything. You just need to satisfy one of these four conditions:

- **SSS (Side-Side-Side)**: All three corresponding sides are equal.
- **SAS (Side-Angle-Side)**: Two sides and the included angle (the angle between them) are equal.
- **AAS (Angle-Angle-Side)**: Two angles and a corresponding side are equal. (Sometimes called ASA).
- **RHS (Right Angle-Hypotenuse-Side)**: In a right-angled triangle, the hypotenuse and one other side are equal.

Suggested Learning Activities for 2.2

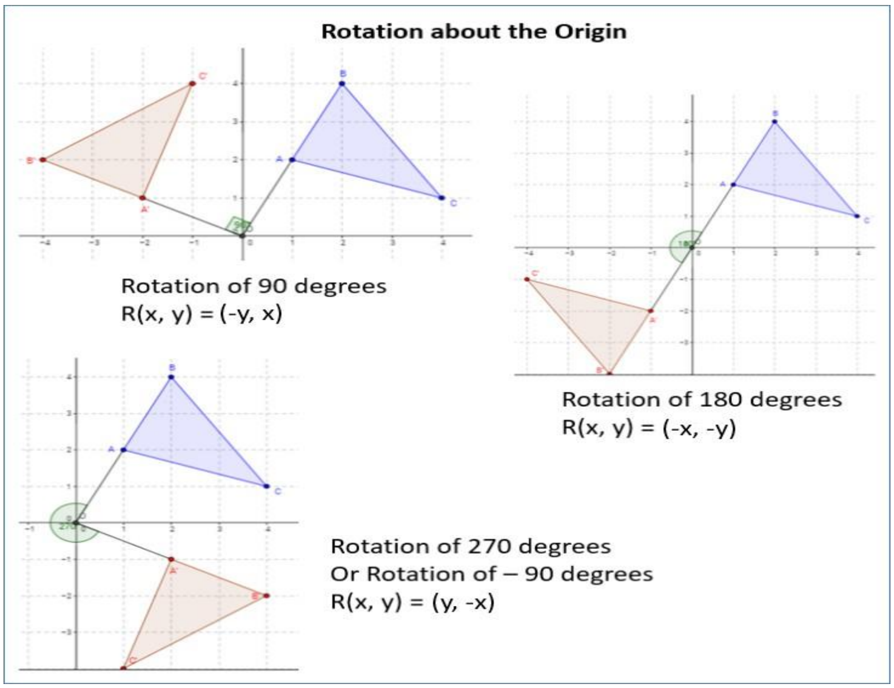
- **Ink Blot Art**: Fold a piece of paper in half. Place wet paint on one side. Fold it over and press. Open it to reveal a perfectly symmetrical image. Discuss the fold line as the "Mirror Line."

- **Congruence Detectives**: Give students a sheet with 20 different triangles. They must cut them out and stack them to find which ones are congruent, then group them by the test used (e.g., "These two matched because of SSS").

SUB STRAND 2.3: ROTATION

Overview

Rotation is a transformation that turns a shape around a fixed point. Unlike reflection, there is no "flipping" involved, just turning.



Properties of Rotation

- **Centre of Rotation**: The pivot point around which the shape turns. It can be inside, on the edge, or outside the shape.
- **Angle of Rotation**: How far the shape turns (e.g., 90 degrees, 180 degrees).
- **Direction of Rotation**:
  - **Clockwise**: Turning in the direction of clock hands (Negative angle in some contexts, but usually specified).
  - **Anticlockwise**: The standard mathematical direction (Positive angle).

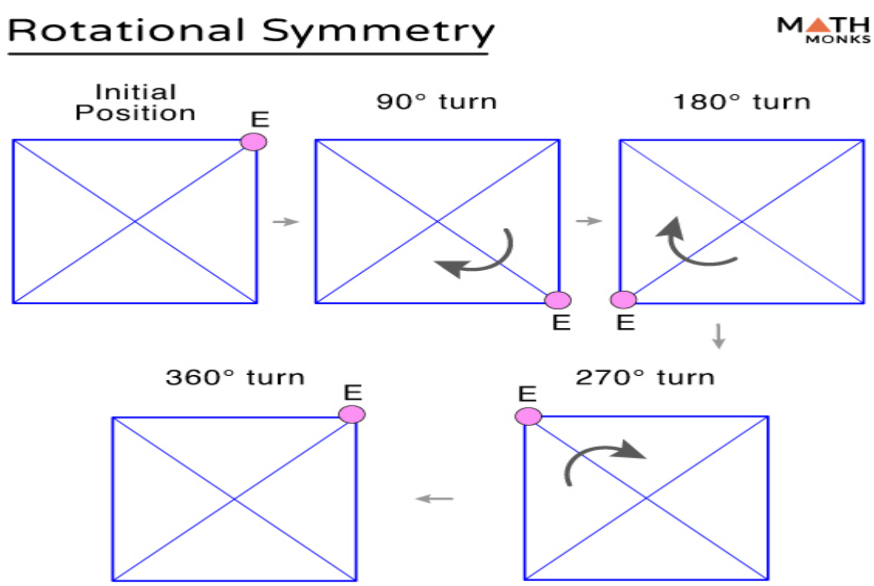
Performing a Rotation

To rotate point P around center C by 90 degrees anticlockwise:

1. Draw a line from C to P.
2. Use a protractor to measure 90 degrees from line CP in the anticlockwise direction.
3. Draw a new line. Measure the distance CP and mark point P' on the new line such that CP = CP'.

Rotational Symmetry

- **Definition**: A shape has rotational symmetry if it looks exactly the same at least once during a full 360-degree turn (other than the start).



- **Order of Rotational Symmetry**: The number of times the shape fits onto itself during one full rotation.

- **Rectangle**: Order 2 (Looks the same at 180 and 360 degrees).
- **Equilateral Triangle**: Order 3 (120, 240, 360 degrees).
- **Square**: Order 4 (90, 180, 270, 360 degrees).
- **Regular Hexagon**: Order 6.

Rotation on the Cartesian Plane

Common rules for rotating about the Origin (0,0):

- **90° Anticlockwise**: Point (x, y) becomes (-y, x).
- **180° Rotation**: Point (x, y) becomes (-x, -y).
- **270° Anticlockwise (or 90° Clockwise)**: Point (x, y) becomes (y, -x).

Finding the Centre of Rotation

If you have an object and its rotated image:

1) Connect a point A to its image A'.
2) Construct the perpendicular bisector of the line AA'.
3) Connect point B to its image B'.
4) Construct the perpendicular bisector of the line BB'.
5) The point where these two bisectors cross is the Centre of Rotation.

Suggested Learning Activities for 2.3

- **Tracing Paper Turning**: Draw a shape on a graph paper. Place tracing paper over it and copy the shape. Put a pin at the origin (0,0). Spin the tracing paper 90 degrees and mark the new coordinates.
- **Symmetry Hunt**: Go outside and find flowers or logos (like the Mercedes symbol) and determine their order of rotational symmetry.

SUB STRAND 2.4: TRIGONOMETRY 1

Overview

Trigonometry is the study of the relationship between the sides and angles of triangles. "Trigonometry 1" focuses on right-angled triangles and is essential for navigation, construction, and physics.

Trigonometric Ratios (SOHCAHTOA)

In a right-angled triangle, we name the sides relative to a specific angle (theta, θ):

- **Hypotenuse**: The longest side, opposite the right angle.
- **Opposite**: The side directly across from angle θ.
- **Adjacent**: The side next to angle θ (that isn't the hypotenuse).

The three primary ratios are:

- **Sine (SOH)**: Sin θ = Opposite / Hypotenuse
- **Cosine (CAH)**: Cos θ = Adjacent / Hypotenuse
- **Tangent (TOA)**: Tan θ = Opposite / Adjacent

Sines and Cosines of Complementary Angles

- **Complementary Angles**: Two angles that add up to 90 degrees. In a right-angled triangle, the two non-right angles are always complementary (A + B = 90).
- **The Rule**: Sin A = Cos (90 - A).
    - The Sine of an angle is equal to the Cosine of its complement.
    - **Example**: Sin 30° = Cos 60°.

- **Example**: Sin 10° = Cos 80°.
- **Why?**: Look at the triangle. The side "Opposite" to angle A is the "Adjacent" side to angle B. So Opp/Hyp for A is the same as Adj/Hyp for B.

Relationship between Sine, Cosine, and Tangent

- **Identity**: Tan θ = Sin θ / Cos θ.
- **Proof**: (Opp/Hyp) ÷ (Adj/Hyp) = (Opp/Hyp) × (Hyp/Adj) = Opp/Adj = Tan.

Trigonometric Ratios of Special Angles

You should memorize the exact values for 30°, 45°, and 60° without a calculator.

- **45° (From an Isosceles Right Triangle 1-1-√2)**:
  - Sin 45° = 1/√2
  - Cos 45° = 1/√2
  - Tan 45° = 1
- **30° and 60° (From an Equilateral Triangle split in half 1-2-√3)**:
  - Sin 30° = 1/2
  - Cos 30° = √3/2
  - Tan 30° = 1/√3
  - Sin 60° = √3/2
  - Cos 60° = 1/2
  - Tan 60° = √3

Applications: Angles of Elevation and Depression

- **Angle of Elevation**: The angle measured upwards from the horizontal line to the object (looking up at a bird).
- **Angle of Depression**: The angle measured downwards from the horizontal line to the object (looking down from a cliff to a boat).
- **Tip**: The angle of depression from the top is equal to the angle of elevation from the bottom (alternate interior angles).

Suggested Learning Activities for 2.4

- **Clinometer Construction**: Make a simple clinometer using a protractor, a straw, a string, and a weight. Use it to measure the angle of elevation to the top of the school flagpole or a tree. Then, measure the distance to the base and use Tangent to calculate the height.
- **Shadow Math**: Measure the length of a student's shadow and the student's actual height. Calculate the angle of elevation of the sun.

SUB STRAND 2.5: AREA OF POLYGONS

Overview

We move beyond basic "base times height" formulas to more advanced techniques for calculating area, especially when the height is not given directly.

Area of a Triangle using Sine

If you know two sides and the *included* angle (the angle sandwiched between them), you can find the area without knowing the perpendicular height.

- **Formula**: Area = 1/2 ab Sin C
  - 'a' and 'b' are the lengths of the two sides.
  - 'C' is the angle between them.
- **Example**: Triangle with sides 5cm and 8cm and angle 30° between them.
  - Area = $0.5 \times 5 \times 8 \times \text{Sin}(30) = 20 \times 0.5 = 10\ \text{cm}^2$.

Heron's Formula (Hero's Formula)

Used when you know all three sides (a, b, c) but no angles and no height.

- **Step 1**: Calculate the semi-perimeter (s).
  - $s = (a + b + c) / 2$
- **Step 2**: Use the formula:
  - Area = $\sqrt{[s(s - a)(s - b)(s - c)]}$
- **Example**: Sides 3, 4, 5.
  - $s = (3+4+5)/2 = 6$.

- Area = $\sqrt{[6(6-3)(6-4)(6-5)]} = \sqrt{[6 \times 3 \times 2 \times 1]} = \sqrt{36} = 6$.

Area of Quadrilaterals

- **Parallelogram**: Area = ab Sin θ (where a and b are adjacent sides and θ is the angle).
- **Trapezium**: Area = 1/2 (a + b)h (where a and b are parallel sides).
- **Kite/Rhombus**: Area = 1/2 × product of diagonals.

Area of Regular Polygons

A regular polygon (like a regular pentagon or octagon) has equal sides and equal angles.

- **Method**: Split the polygon into identical isosceles triangles meeting at the center.
  - For an n-sided polygon, you get n triangles.
  - The central angle of each triangle is 360/n.
- **Calculation**: Find the area of one triangle using (1/2 ab Sin C) and multiply by n.
- **Example**: Regular hexagon with radius (distance from center to corner) 10cm.
  - 6 triangles.
  - Angle = 360/6 = 60°.
  - Area of one = 1/2 × 10 × 10 × Sin 60 = 50 × 0.866 = 43.3.
  - Total Area = 43.3 × 6 = 259.8 cm^2.

Suggested Learning Activities for 2.5

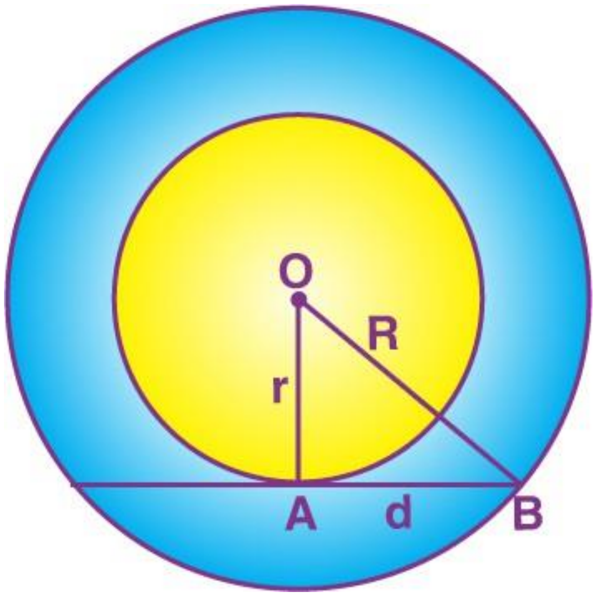
- **The Land Surveyor**: Stake out a non-right-angled triangular plot on the school field. Measure the three sides using a tape measure. Calculate the area using Heron's formula. Then measure an angle and use the Sine rule area to check if they match.
- **Polygon Pizza**: Cut a paper hexagon into 6 equilateral triangles to demonstrate the area summation method.

SUB STRAND 2.6: AREA OF A PART OF A CIRCLE

Overview

This section focuses on finding the areas of specific "slices" or "chunks" of a circle.

Area of an Annulus



- **Definition**: An annulus is the ring-shaped region between two concentric circles (circles with the same center). Think of a washer or a CD.
- **Formula**: Area = Area of Outer Circle - Area of Inner Circle.
  - Area = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$
  - Where R is the outer radius and r is the inner radius.

Area of a Sector

- **Definition**: A "slice of pie." A region bounded by two radii and an arc.
- **Formula**: Area = $(\theta / 360) \times \pi r^2$
  - Where θ is the angle of the sector at the center.

- **Logic**: A full circle is 360°. If you have 60°, you have 60/360 or 1/6th of the circle.

Area of a Segment

- **Definition**: The region bounded by a chord and an arc. It's the "crust" of the pizza slice if you cut off the triangular tip.
- **Formula**: Area of Segment = Area of Sector - Area of Triangle.
    - Step 1: Calculate Sector Area = $(\theta/360)\pi r^2$.
    - Step 2: Calculate Triangle Area = $1/2\ r^2 \sin\theta$ (since the two sides are both radii).
    - Step 3: Subtract Triangle from Sector.

Area of Common Regions (Intersecting Circles)

When two circles overlap, they form a lens shape or "vesica piscis."

- **Method**:
    1. Draw a chord connecting the intersection points.
    2. This splits the shape into two segments (one from each circle).
    3. Calculate the area of the segment for circle 1.
    4. Calculate the area of the segment for circle 2.
    5. Add them together.

Suggested Learning Activities for 2.6

- **Dartboard Design**: Analyze a dartboard. It consists of many annuluses (rings) and sectors. Calculate the area of the "Triple 20" region.

- **Wiper Blade Math**: Measure the length of a car wiper blade and the angle it sweeps. Calculate the area of the windscreen cleaned by the wiper (which is usually an annulus sector).

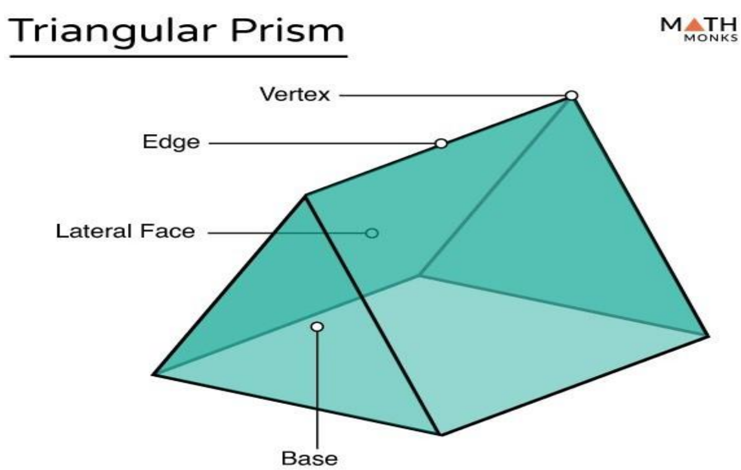
SUB STRAND 2.7: SURFACE AREA AND VOLUME OF SOLIDS

Overview

Here we enter the 3D world. Learners will calculate the space occupied by solids (volume) and the area of their outer skins (surface area).

Prisms

- **Definition**: A solid with a uniform cross-section (e.g., cylinder, triangular prism, cuboid).
- **Volume**: Area of Cross-Section × Length.
- **Surface Area**: sum of the areas of all faces.



Pyramids and Cones

These solids taper to a point (apex).

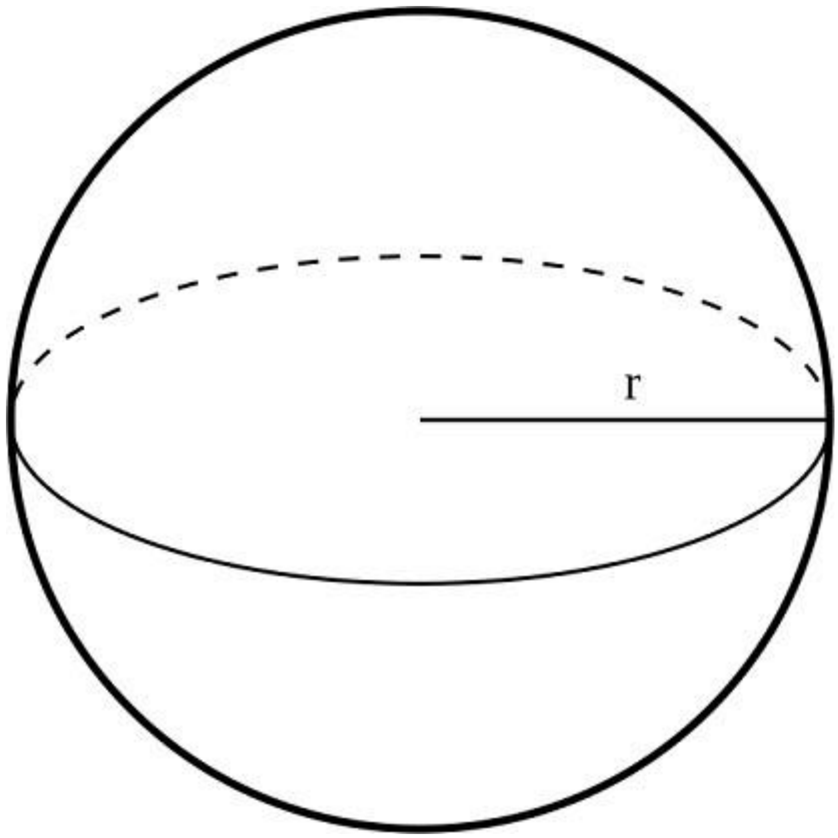
- **Volume**: 1/3 × Base Area × Perpendicular Height.
  - Cone Volume: 1/3 $\pi r^2 h$
  - Pyramid Volume: 1/3 (Length × Width) × h

- **Surface Area of Cone**:
    - Curved Surface Area = πrl (where l is the slant height).
    - Total Surface Area = $\pi rl + \pi r^2$ (base).
    - **Tip**: Find slant height 'l' using Pythagoras: $l^2 = r^2 + h^2$.

CONE

PYRAMID

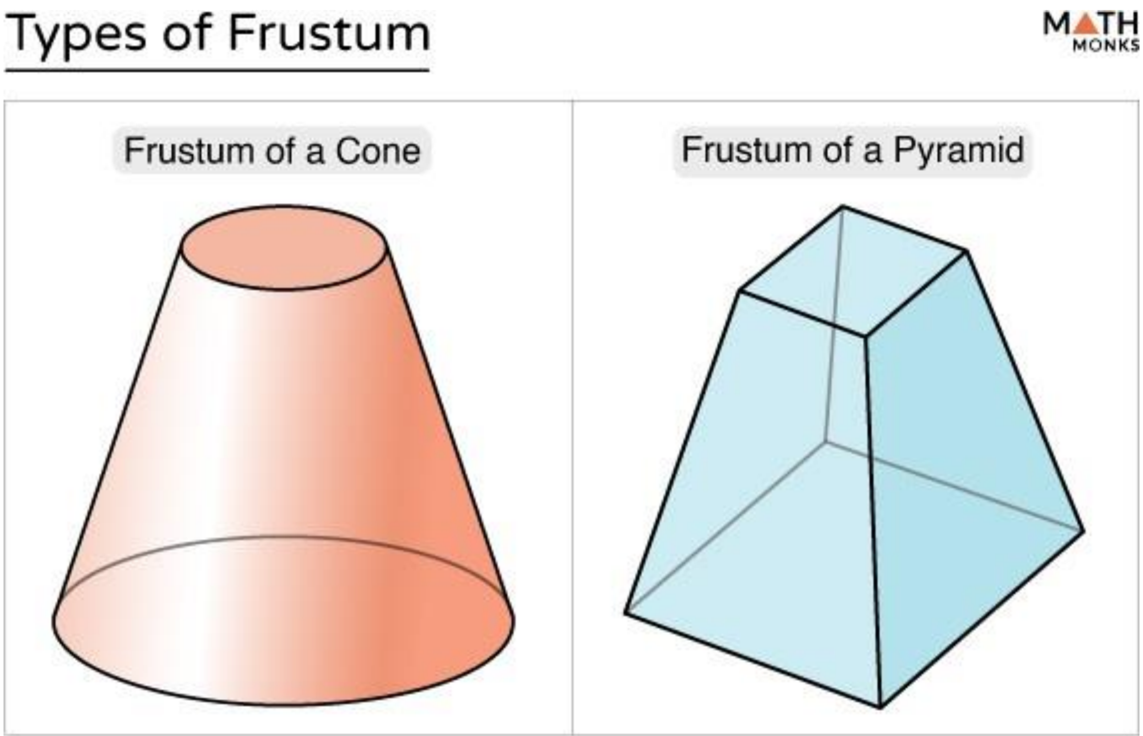
Spheres



- **Volume**: $V = 4/3\ \pi r^3$
- **Surface Area**: $SA = 4\ \pi r^2$
- **Hemisphere (Half sphere)**:
    - Volume = $2/3\ \pi r^3$
    - Curved Surface Area = $2\ \pi r^2$
    - Total Surface Area (solid) = $3\ \pi r^2$ ($2\pi r^2$ for the dome + πr^2 for the flat circular base).

Frustums

A frustum is the bottom part of a cone or pyramid after the top has been sliced off parallel to the base (like a bucket).



- **Volume Method 1 (Subtraction)**:
  - Imagine the full cone existed. Calculate its volume.
  - Calculate the volume of the small cone that was cut off.
  - Frustum Volume = Volume of Big Cone - Volume of Small Cone.
  - **Finding the height of the missing cone**: Use similarity.

    Radius(big)/Radius(small) = Height(big)/Height(small).
- **Surface Area**:
  - Calculate Curved Surface Area of the big cone (πRl).
  - Subtract Curved Surface Area of the small cone (πrl).

- Add the area of the top circular face (πr^2) and bottom circular face (πR^2).

Composite Solids

Solids made of multiple shapes joined together (e.g., a silo is a cylinder + a hemisphere).

- **Volume**: Simply add the volumes of the individual parts.
- **Surface Area**: Add the exposed areas. **Caution**: Do not include the internal faces where the shapes are joined. For a silo, you would add the curved area of the cylinder and the curved area of the hemisphere, plus the bottom circle of the cylinder. You would NOT add the top circle of the cylinder or the base of the hemisphere because they are inside the solid.

Suggested Learning Activities for 2.7

- **Packaging Design**: Bring in various containers (Toblerone box, Pringles can, ice cream cone). Disassemble them to see their "nets" (2D patterns). Calculate the amount of cardboard used (Surface Area) vs. the amount of product they hold (Volume).
- **The Bucket Challenge**: Measure a bucket (frustum). Calculate how many liters of water it can hold using the frustum volume method. Fill it with water using a measuring jug to verify.

SUB STRAND 2.8: VECTORS I

Overview

Vectors introduce the concept of "direction" to numbers. This is critical for physics (force, velocity) and computer graphics.

Scalars vs. Vectors

- **Scalar Quantity**: Has magnitude (size) only.
  - Examples: Time, Mass, Distance, Temperature, Speed.
- **Vector Quantity**: Has both magnitude and direction.
  - Examples: Displacement, Velocity, Force, Acceleration.

Vector Notation

- **Graphical**: An arrow. The length represents magnitude; the arrowhead shows direction.
- **Written**: A letter with a line or tilde under or above it (e.g., **a** or <u>a</u>), or directed line segment notation AB with an arrow on top.

Column Vectors

- Represented as a vertical matrix: (x / y).
- **x**: Movement in the horizontal direction (right is positive, left is negative).
- **y**: Movement in the vertical direction (up is positive, down is negative).

Magnitude of a Vector

- The "length" of the vector.
- If vector **a** = (x / y), then Magnitude |**a**| = $\sqrt{(x^2 + y^2)}$.

- This is essentially the Pythagorean theorem.

Operations on Vectors

- **Addition**: Add the corresponding components.
  - (2 / 3) + (4 / -1) = (6 / 2).
  - **Triangle Law**: To add vector AB and BC, you travel from A to B, then B to C. The result is the direct path AC.
  - **Parallelogram Law**: If two vectors start from the same point, their sum is the diagonal of the parallelogram formed by them.
- **Scalar Multiplication**: Multiplying a vector by a number.
  - 2 × (3 / -2) = (6 / -4).
  - This makes the vector longer (or shorter) but keeps the direction parallel. A negative scalar reverses the direction.

Position Vectors

- A vector that describes the position of a point relative to the Origin (0,0).
- If point P is at (3, 4), the position vector **OP** is (3 / 4).

Translation

A translation slides a shape. It is described by a translation vector.

- If point A (2, 1) is translated by vector T = (3 / 2), the new point A' is (2+3, 1+2) = (5, 3).

Suggested Learning Activities for 2.8

- **Treasure Hunt**: Create a map of the school yard. Give instructions in vectors: "Start at the tree. Move (10m / 0m), then (0m / -5m)." Students must follow the vectors to find the hidden flag.
- **Vector Tug-of-War**: Draw vectors on the board representing forces pulling on an object. Use the parallelogram law to find the "Resultant Force" and predict which way the object will move.

SUB STRAND 2.9: LINEAR MOTION

Overview

This sub-strand applies mathematical graphs to the physics of motion. It deals with objects moving in a straight line.

Key Concepts

- **Distance**: Total ground covered (Scalar).
- **Displacement**: Distance from the start point in a specific direction (Vector).
- **Speed**: How fast an object moves (Distance / Time).
- **Velocity**: Speed in a given direction (Displacement / Time).
- **Acceleration**: Rate of change of velocity (Change in Velocity / Time).

Displacement-Time Graphs

- Plots Displacement (y-axis) vs. Time (x-axis).
- **Gradient (Slope)**: Represents Velocity.
    - Steep slope = Fast velocity.
    - Flat line (horizontal) = Stationary (Not moving).
    - Negative slope = Returning to start.

Velocity-Time Graphs

- Plots Velocity (y-axis) vs. Time (x-axis).
- **Gradient (Slope)**: Represents Acceleration.
    - Positive slope = Accelerating (speeding up).
    - Flat line = Constant velocity (Zero acceleration).

  - Negative slope = Decelerating (slowing down).
- **Area under the graph**: Represents the Distance Traveled.
  - Split the area into rectangles and triangles to calculate the total distance.

Relative Speed

- **Bodies moving in opposite directions**: They pass each other very fast.
  - Relative Speed = Speed A + Speed B.
- **Bodies moving in the same direction**: One catches up slowly.
  - Relative Speed = Speed A - Speed B.

Equations of Linear Motion (Kinematic Equations)

While graphing is the focus, understanding the relationships is key:

- $v = u + at$ (Velocity = Initial Velocity + Acceleration × Time)
- $s = ut + 1/2\ at^2$ (Displacement)
- $v^2 = u^2 + 2as$

Suggested Learning Activities for 2.9

- **The 100m Dash**: Have students run 100m and record their times at 10m intervals. Plot the Distance-Time graph. Analyze the curve to see where they were fastest (steepest slope) and where they started to tire (slope flattening).

- **Traffic Analysis**: Watch a video of cars. Discuss relative speed. If you are doing 80km/h and a car passes you doing 100km/h, it looks like it is only moving at 20km/h relative to you.

Conclusion

Strand 2.0 builds a robust toolkit for understanding the physical world. From the micro-scale of similar cells to the macro-scale of planetary volumes and vehicle motion, these concepts connect abstract numbers to tangible reality. Mastery of these sub-strands ensures a solid foundation for Physics, Engineering, and Design.

STRAND 3.0: STATISTICS AND PROBABILITY

Introduction

Welcome to the world of data and chance! In this strand, you will learn how to make sense of the vast amount of information (data) we encounter every day and how to predict the likelihood of events occurring. This knowledge is essential for careers in data science, business, engineering, and government policy.

SUB-STRAND 3.1: STATISTICS I

Overview

Statistics is the branch of mathematics that deals with collecting, organizing, analyzing, representing, and interpreting data. In this section, we move from simple lists of numbers to complex grouped data and visual graphs.

Part A: Introduction to Data and Collection

1. What is Data?

Data refers to raw facts, figures, numbers, or descriptions collected for reference or analysis.

- **Qualitative Data:** Describes qualities or characteristics (e.g., eye color, favorite food, gender). It is descriptive.
- **Quantitative Data:** Measures values or counts and is expressed as numbers. It can be further divided into:

  - **Discrete Data:** Can only take specific, exact values. It is usually counted (e.g., number of students in a class, shoe size, number of cars in a parking lot). You cannot have 2.5 students.
  - **Continuous Data:** Can take any value within a range. It is usually measured (e.g., height, weight, time, temperature). You can be 165.5 cm tall.

2. Sources of Data

- **Primary Data:** Information collected directly by you (the researcher) for a specific purpose.
  - *Examples:* Measuring the height of your classmates, counting vehicles passing the school gate.
  - *Pros:* Accurate and specific to your needs.
  - *Cons:* Time-consuming and expensive.
- **Secondary Data:** Information that has already been collected by someone else.
  - *Examples:* Census reports, weather records from the internet, school attendance records from the principal's office.
  - *Pros:* Cheap and easy to obtain.
  - *Cons:* May be outdated or not exactly what you need.

3. Methods of Data Collection

To gather data, we use several standard methods:

- **Observation:** Watching and recording behavior or events (e.g., observing traffic flow).
- **Interview:** Asking questions face-to-face or over the phone.
- **Questionnaire:** Giving people a list of written questions to answer.

- **Experiment:** Performing a controlled test (e.g., rolling a die 100 times).

Tally Charts

When collecting data, we often use a tally chart to keep count.

- A vertical mark (|) represents one item.
- Every fifth item is a diagonal line across the previous four (an enclosed bundle), making it easy to count in fives.

Tip: Always determine if your data is discrete or continuous before you start organizing it, as this affects how you draw your graphs later!

Part B: Frequency Distribution Tables

Raw data is often messy. To make sense of it, we organize it into **Frequency Distribution Tables**.

1. Ungrouped Data

This is used when the data set is small or the range of values is small. We list each specific value.

Example: Rolling a Die

Imagine you roll a die 20 times and get these results:

1, 3, 2, 5, 6, 1, 3, 3, 4, 2, 1, 5, 3, 6, 2, 4, 3, 1, 5, 2

Frequency Table:

Score (x)	Tally	Frequency (f)

1		
2		
3		
4		
5		
6		
Total		**20**

2. Grouped Data

When you have a lot of data or a wide range of values (like marks out of 100 for 50 students), listing every number is impossible. We group the data into **Classes**.

Key Terminology for Grouped Data:

- **Class Interval:** The group of values (e.g., 10 – 19).
- **Class Limits:** The smallest and largest values that can belong to a class.
  - *Lower Class Limit:* 10
  - *Upper Class Limit:* 19
- **Class Boundaries:** These are the true mathematical edges of the class, used to close the gap between classes. To find them, subtract 0.5 from the lower limit and add 0.5 to the upper limit (for whole numbers).
  - *Lower Class Boundary:* 9.5
  - *Upper Class Boundary:* 19.5
  - *Why?* It ensures that a value like 19.5 doesn't fall into a "gap" between the 10-19 class and the 20-29 class.

- **Class Width (or Size):** The difference between the Upper Boundary and the Lower Boundary.
    - *Formula:* Class Width = Upper Boundary − Lower Boundary.
    - *Example:* 19.5 − 9.5 = 10.
- **Class Mark (Midpoint):** The center of the class. This value represents the whole class when calculating the mean.
    - *Formula:* Class Midpoint = (Lower Limit + Upper Limit) ÷ 2.
    - *Example:* (10 + 19) ÷ 2 = 14.5.

Example Scenario:

Marks of 40 students:

12, 19, 22, 25, 28, 31, 33, 33, 35, 39, 40, 41, 42, 44, 45, 48, 49, 50, 51, 52... (and so on).

Grouped Frequency Table:

Marks (Class)	Class Boundaries	Class Midpoint (x)	Frequency (f)
10 – 19	9.5 – 19.5	14.5	2
20 – 29	19.5 – 29.5	24.5	3
30 – 39	29.5 – 39.5	34.5	5
40 – 49	39.5 – 49.5	44.5	8

Part C: Measures of Central Tendency

Central tendency helps us find a single value that describes the "center" or "typical" value of the entire set of data.

1. Mean (The Average)

Symbol: $\bar{x}$ (read as "x-bar")

A) For Ungrouped Data:

Add all the values and divide by the number of values.

- **Formula:** $\bar{x} = (\Sigma x) / n$
  - *Where Σ (Sigma) means "sum of"*
  - *x is the values*
  - *n is the number of items*

B) For Grouped Data:

Since we don't know the exact raw numbers inside the groups, we use the Class Midpoint (x) to represent them.

1. Find the midpoint (x) for each class.
2. Multiply the frequency (f) by the midpoint (x) to get **fx**.
3. Sum the fx column (Σfx).
4. Sum the frequency column (Σf).
5. Divide.

- **Formula:** $\bar{x} = (\Sigma fx) / (\Sigma f)$

Worked Example (Grouped Mean):

Class	Midpoint (x)	Frequency (f)	f × x
0 – 10	5	4	20

10 – 20	15	6	90
20 – 30	25	10	250
Totals		**Σf = 20**	**Σfx = 360**

- **Mean** = 360 ÷ 20 = **18**

2. Mode (The Most Common)

A) For Ungrouped Data:

The number that appears most frequently.

- *Example:* 2, 4, 4, 4, 7. Mode = 4.

B) For Grouped Data (Modal Class):

We look for the Modal Class—the class interval with the highest frequency.

- *Example:* In the table above, the class 20–30 has the highest frequency (10). Therefore, the Modal Class is 20–30.

3. Median (The Middle Value)

A) For Ungrouped Data:

Arrange data in ascending order. The value in the middle is the median.

- If *n* is odd: The position is (n + 1) / 2.
- If *n* is even: The median is the average of the two middle numbers.

B) For Grouped Data (Median Class):

We need to find the class where the middle student is located.

1. Calculate total frequency (N).
2. Find the position N/2.
3. Use **Cumulative Frequency** to locate which class contains that position.

Cumulative Frequency: The running total of frequencies.

- *Example:* If Class 1 has 4 students, and Class 2 has 6 students, the cumulative frequency at the end of Class 2 is 4 + 6 = 10.

Tip: When calculating the mean for grouped data, accuracy depends on your arithmetic. Always double-check your "fx" multiplication!

Part D: Representation of Data

Visualizing data makes it easier to understand trends.

1. Histograms

A histogram looks like a bar chart, but there are crucial differences:

- **No Gaps:** The bars touch each other because the data is continuous.
- **X-Axis:** Uses **Class Boundaries**, not class limits.
- **Area:** The area of the bar represents the frequency.

How to Draw a Histogram:

1. **Equal Class Widths:**
    - Plot Class Boundaries on the horizontal axis (x-axis).
    - Plot Frequency on the vertical axis (y-axis).
    - Draw bars so they touch.

2. **Unequal Class Widths (Advanced Tip):**
   - Sometimes classes are different sizes (e.g., 0-10, 10-15, 15-30).
   - You **cannot** use Frequency on the y-axis. You must calculate **Frequency Density**.
   - **Formula:** Frequency Density = Frequency ÷ Class Width.
   - Plot Frequency Density on the y-axis.

[Image Suggestion: Search for "Histogram with equal class widths example" to see how the bars touch at the boundaries 9.5, 19.5, etc.]

2. Frequency Polygons

A frequency polygon is a line graph that connects the midpoints of the tops of the histogram bars.

Steps to Draw:

1. Calculate the **Midpoint** of each class.
2. Plot the Midpoint on the x-axis against the Frequency on the y-axis.
3. Join the points with straight lines (use a ruler).
4. **Closing the Polygon:** To make it a closed shape, imagine a class before the first one and a class after the last one (with 0 frequency) and connect the line to the x-axis at these theoretical midpoints.

Part E: Interpretation of Data

Once a graph is drawn, you must be able to read it to make informed decisions.

- **Shape:** Is the data symmetrical (bell-shaped)? Is it skewed (leaning) to the left or right?

- **Spread:** Are the marks close together or widely spread out?
- **Outliers:** Are there any values that are extremely high or low compared to the rest?

Real-Life Application:

A school principal looks at a histogram of math scores.

- If the "peak" is at low marks, the exam might have been too hard, or students need more help.
- If the graph is flat (no peak), the students have very varied abilities.

Activity 3.1: The Height Survey

1. **Collect:** Measure the height of every student in your class (in cm).
2. **Organize:** Create a grouped frequency table with class widths of 5cm (e.g., 150-154, 155-159).
3. **Calculate:** Find the Mean height and the Modal Class.
4. **Represent:** Draw a Histogram and a Frequency Polygon on graph paper.
5. **Analyze:** Write one sentence describing the height of the "average" student.

SUB-STRAND 3.2: PROBABILITY 1

Overview

Probability is the study of chance. It measures how likely it is that an event will happen. We use it in weather forecasting, sports, insurance, and gaming.

Part A: Basic Concepts

1. Key Definitions

- **Experiment:** An activity involving chance that produces results (e.g., tossing a coin, rolling a die).
- **Outcome:** A possible result of an experiment (e.g., getting a "Head").
- **Event:** A specific outcome or collection of outcomes you are interested in (e.g., getting an even number).
- **Sample Space (S):** The list of *all* possible outcomes.
    - *Example for a die:* S = {1, 2, 3, 4, 5, 6}.

2. The Probability Scale

Probability is always a number between 0 and 1.

- **0:** Impossible (e.g., Humans growing wings naturally tomorrow).
- **0.5 (or 50%):** Even chance (e.g., Tossing a coin).
- **1:** Certain (e.g., The sun will rise in the East).

Notation:

P(Event) = Probability of that event happening.

- P(A) is the probability of Event A.

Part B: Experimental vs. Theoretical Probability

1. Theoretical Probability

This is what *should* happen based on math.

- **Formula:** P(Event) = (Number of favorable outcomes) / (Total number of possible outcomes)
- *Example:* Rolling a die. What is the probability of getting a 5?
    - Favorable outcomes: 1 (only the number 5).
    - Total outcomes: 6.
    - P(5) = 1/6.

2. Experimental Probability

This is what *actually* happens when you do the experiment.

- **Formula:** P(Event) = (Number of times event happened) / (Total number of trials)
- *Example:* You toss a coin 10 times. It lands on Heads 7 times.
    - Experimental P(Head) = 7/10 or 0.7.
    - Theoretical P(Head) should be 0.5.
- **Law of Large Numbers:** The more times you repeat an experiment (e.g., tossing the coin 1000 times), the closer the Experimental Probability gets to the Theoretical Probability.

Part C: Probability Space & Combined Events

When we have two experiments happening at once (e.g., tossing two coins), we need to list the **Probability Space** to see all outcomes.

Example: Tossing Two Coins

Outcomes for Coin 1: H, T

Outcomes for Coin 2: H, T

Outcomes List:

1. HH (Head, Head)
2. HT (Head, Tail)
3. TH (Tail, Head)
4. TT (Tail, Tail)

Total outcomes = 4.

- P(Two Heads) = 1/4.
- P(One Head and One Tail) = 2/4 = 1/2.

[Image Suggestion: Search for "Outcomes grid for rolling two dice" to see the 36 possible outcomes matrix]

Part D: Types of Events

1. Mutually Exclusive Events

Two events that **cannot** happen at the same time.

- *Example:* Turning left or turning right. You cannot do both at once.
- *Example:* Rolling a 2 and Rolling a 5 on a single die throw.
- **The "OR" Rule (Addition Law):**
    - If outcomes are mutually exclusive, P(A **or** B) = P(A) + P(B).
    - *Question:* What is the probability of rolling a 2 **or** a 5?
    - *Answer:* P(2) + P(5) = 1/6 + 1/6 = 2/6 = 1/3.

2. Independent Events

Two events where the result of the first one **does not affect** the result of the second one.

- *Example:* Tossing a coin and rolling a die. Getting a "Head" doesn't change the die's chance of rolling a "6".
- **The "AND" Rule (Multiplication Law):**
    - If outcomes are independent, P(A **and** B) = P(A) × P(B).
    - *Question:* What is the probability of getting a Head **and** a 6?
    - *Answer:* P(Head) × P(6) = 1/2 × 1/6 = 1/12.

Part E: Tree Diagrams

A **Tree Diagram** is a branching diagram that shows all possible outcomes of combined events. It is the best tool for solving complex probability questions.

How to Draw a Tree Diagram:

Scenario: A bag has 3 Red balls and 2 Blue balls. You pick one, put it back (replacement), and pick another.

1. **Start Point:** Draw two branches (one for Red, one for Blue).
2. **Label Branches:** Write the probability on each branch.

    - Red = 3/5
    - Blue = 2/5

3. **Second Stage:** From the end of the Red branch, draw two more branches (Red, Blue). Do the same for the Blue branch.
4. **Calculate Outcomes:** Multiply along the branches to get the final probability for that path.

The Math (With Replacement):

- **Path Red-Red:** 3/5 × 3/5 = 9/25
- **Path Red-Blue:** 3/5 × 2/5 = 6/25
- **Path Blue-Red:** 2/5 × 3/5 = 6/25
- **Path Blue-Blue:** 2/5 × 2/5 = 4/25

Check: Add all final fractions. They must equal 1. (9+6+6+4)/25 = 25/25 = 1. Correct!

Tip: "With replacement" means the total number of items stays the same (denominator is constant). "Without replacement" means the total decreases by 1 for the second pick.

Part F: Application in Real Life

- **Weather:** A 30% chance of rain means in similar conditions in the past, it rained 30 times out of 100.
- **Business:** Insurance companies use probability to calculate premiums based on the risk of an accident.
- **Sports:** Calculating the probability of a team winning based on past performance stats.

Activity 3.2: The Fair Game Design

1. **Design:** Create a simple game using a coin and a die.
   - *Example Rule:* To win, you must toss a Head AND roll a number greater than 4.
2. **Calculate:** Work out the **Theoretical Probability** of winning your game.
   - P(Head) = 1/2
   - P(>4) = P(5 or 6) = 2/6 = 1/3
   - P(Win) = 1/2 × 1/3 = 1/6.
3. **Experiment:** Play the game 30 times. Record how many times you actually won.
4. **Compare:** Calculate your Experimental Probability and compare it to your Theoretical calculation. Are they close?

REVIEW: Summary of Formulas

Statistics:

- **Class Width:** Upper Boundary - Lower Boundary
- **Midpoint (x):** (Lower Limit + Upper Limit) / 2
- **Mean (Grouped):** $\bar{x} = (\Sigma fx) / (\Sigma f)$
- **Frequency Density:** Frequency / Class Width

Probability:

- **Range:** $0 \leq P(A) \leq 1$
- **Theoretical:** Favorable / Total
- **Experimental:** Successes / Trials

- **OR Rule (Mutually Exclusive):** P(A) + P(B)
- **AND Rule (Independent):** P(A) × P(B)

Tips for Success in Grade 10 Math:

1. **Be Neat:** Statistics requires tables. Use a ruler. If your columns are messy, you will add the wrong numbers.
2. **Read Carefully:** Does the question say "inclusive" or "exclusive"? Does it say "with replacement" or "without"? One word changes the whole math.
3. **Sanity Check:** If you calculate a probability of 1.5, you are wrong (it must be 0-1). If you calculate the mean height of a student is 15 meters, you are wrong. Always ask: "Does this answer make sense?"